The subject of math, as used in music, isn't one that comes up a lot in conversation. In fact, it's usually the music student, maybe some teachers, definitely musicologists and sometimes certain composers who are interested, and then usually in an academic way. Being, for the most part, a serial composer (more generally, atonal), the subject interested me from a music construction perspective. I wanted to know how math helped define the structural elements of a work, like pitch organization, rhythm and overall form.

I read the usual treatises[1] and other writings; found them exceedingly complex, then went back to what I was already doing. I was already basing much of what I was doing on mathematical schemes, usually those inherent in typical musical systems. I did nothing overtly to establish my own mathematical ideas. But I saw where many of the serialists of the 50s and 60s used more complex mathematical schemes to make better use of the series, especially its integer notation component. Even though the resulting total serialism self-destructed under its own weight, many of the math schemes it used were still intriguing to me.

I felt it necessary to better understand these schemes and determine if I could apply them to my own work. In that spirit, I've sort of re-discovered *Iannis Xenakis* through two books[2] I recently acquired. The most striking aspect of his work is how much of it is derived from high-level mathematics, especially as it pertains to architecture. That's because *Xenakis* was first an architect, working for *Le Corbusier*, and then a composer. His own book[3] *Formalized Music (Thought and Mathematics in Music)* is a very in-depth look at how much math played a role in his music. I assure you, it's mind-boggling. If you have math anxiety, this will freak you out.

The other composer who used math extensively, including his own application of set theory[4], was *Milton Babbitt*[5]. He may not have gone as far as *Xenakis*, but he certainly used math more than many of the other serialists of his day. In delving into his writings, I was admittedly overwhelmed, not always getting it right on the first reading. It was through the writings of others, like *Wuorinen*[6], who put things in easier-to-understand terms, that the concepts finally came alive for me.

I have a healthy respect for math in general, and more so as it applies to music. But I am not a mathematician by any stretch of the imagination. Calculators and computers are my friends, when it comes to figuring these things out. But, as a composer, I felt I owed it to myself to gain a better understanding of how these composers used math in their music. That's why I embarked on this journey to sort through *Xenakis* and *Babbitt*, as well as others who have both written on and applied the subject in their work.

Math does *not* play a role in the act of real-time composing. It has more to do with pre-composing, or whatever term best describes the processes one goes through to develop materials to be used in a composition, before actually starting to compose it. This includes developing twelve tone rows, as well as any other tonal organization schemes, as *Elliot Carter* did. But it also pertains to predetermining the overall form of a piece, which *Xenakis* did frequently, and rhythmic structures.

Applying mathematic schemes to a composition can be a double-edged sword. If you rely on it too heavily, your piece will reflect that in how it sounds; often making it sound machine-made; a common complaint about some minimalist music. I believe the final arbiter in determining if a piece is worthy or not is how it sounds. All music evokes some kind of emotional response in listeners. I personally believe there's nothing at all wrong with that. It's what draws someone in to listen, or turns them off, depending on a myriad of factors.

I always audit and edit my work before publishing it to determine if it sounds okay to me. As you're composing, you tend to get caught up in the piece's construction and don't always pay enough attention to how it sounds in its entirety. I will often modify things if it tends to make the piece sound unacceptable for any reason. I figure if I like it, others will too. Conversely, if it doesn't sound right to me, others may agree. The schemes and their mathematical basis is not the end game; how the piece sounds always is, as far as I'm concerned.

That being said, however, I believe that how a piece is constructed, whether mathematically or intuitively, is important. Believing that musical intuition is more genuine and, therefore, more real than subjecting music to mathematical constructs, is debatable. In spite of what we may egotistically believe, our intuition can be flawed. There's no guarantee that the music we're inspired to write will turn out wonderful. Often it sounds far crappier than we first heard in our heads, as we were conceiving it.

Without realizing it, we can subconsciously fall into familiar patterns that we've assimilated over the years. We have a lot of music stuck in our heads and sometimes it bleeds through. When that happens, the music that results can be as monotonous or banal as we might assume more mathematically structured music to be. Those who refer to the bad old days of Mozart and Hayden as being both less complicated, yet still awesome and inspiring, are missing the point.

Take the Sonata form[7]. There is a very definite structure to it that most composers, especially those from the bad old days, followed closely. You have the intro, the exposition, followed by the development, ending with the recapitulation. There are guidelines for theme development and even tempo. Basically, there's a lot of pre-composition going on with a sonata; it's more like a template to follow, and all you essentially do is fill in the blanks.

The string quartet is another classic form that has undergone dramatic changes over the years. All of these forms have a foundation in mathematics to some extent; both simple and complex. I think the question for any composer is how deep do they want to dig into the more advanced mathematical structures and schemes involved.

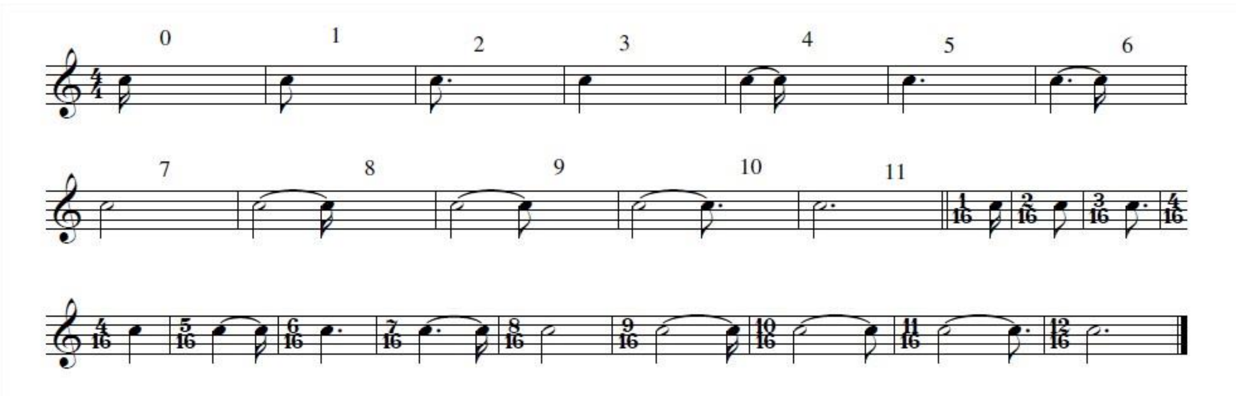
Most composers are not math-savvy and feel pursuing it further is too much of a hassle. They're comfortable composing the way they've been and don't believe it's worth the extra work. Some, who make at least some of their living writing music, are probably already into a writing style, and exploring the math thing would do nothing for them; in fact, it may be more of a hindrance because they usually deal with clients and deadlines, and don't have the time to experiment.

I get that. My situation is entirely different. I don't do this for money, I do it because it's my passion. I'm retired and get a modest pension that keeps me in groceries and pays the mortgage. I have the time and the inclination to experiment, so exploring mathematics and learning to apply it to my music doesn't get in the way of anything.

The point of this paper is to acknowledge that higher math has a role to play in the structure of much of modern music, and math generally is inherent in all music, regardless of what era it's from. It's also about me relating my experiences pursuing musical math and which schemes I found the most interesting. I use those frequently. The other thing I learned is not to over-use them. They can turn an interesting piece into one that turns the listener off.

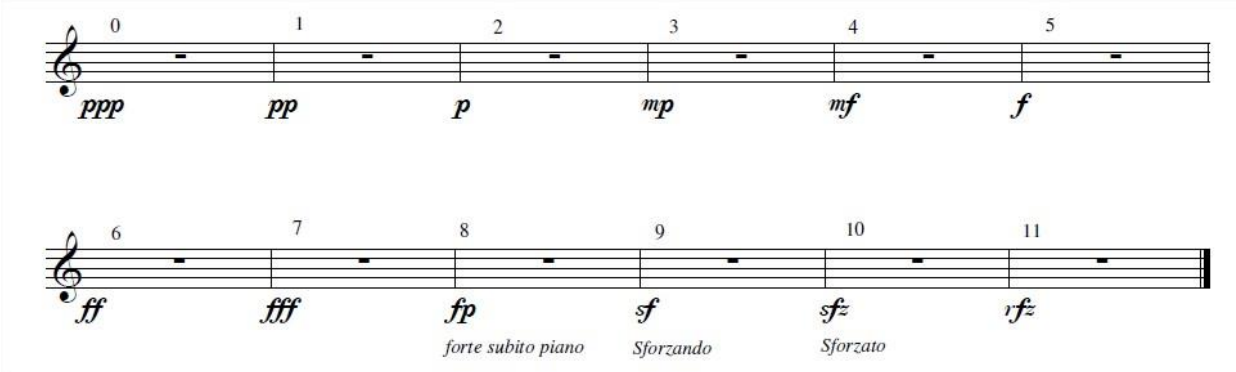
I use them sparingly and try to seamlessly blend them into the rest of the piece, so they don't jump out as blatantly different. The continuity of a piece is, I feel, important to maintain. Even pieces that are essentially shorter segments linked together, I still transition from one segment to the next, so that there's a contiguous feel to it. So I'm careful where I use these schemes and their duration.

Here's an example of a duration scheme based on multiples of a sixteenth note. Each integer, starting with 0 (being one sixteenth note), increments one sixteenth note through to 11, which is a dotted half note.



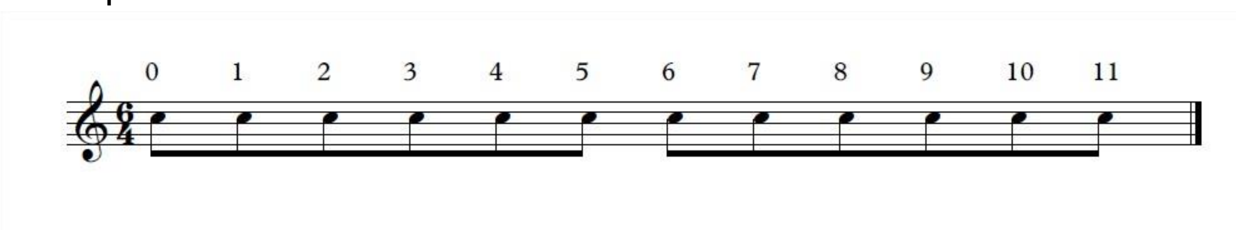
The integers used in a twelve note series are determined as follows.

The first note of the series is always zero (0). The next integer is the number of semitones from the first, in our example it's six (6). Each note after that also represents the number of semitones from the first note in the series (0). This repeats until all twelve notes are stated. These integers are what we use in applying whatever math scheme we devise. Besides duration, there's also dynamics. Here's an example.

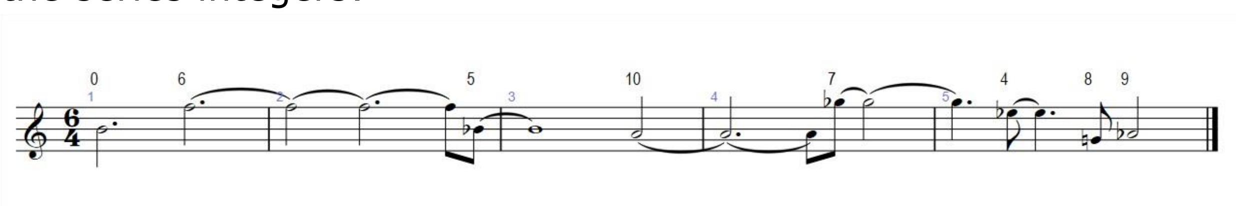


I rarely use the integers-as-dynamics scheme because I find it too difficult to discern the subtle changes, especially in a piece of a faster tempo. But it shows how the you can serialize any parameter, even though from a practical standpoint, it isn't always a good idea.

The other scheme I sometimes use is one developed by *Milton Babbitt*. It's called the *Time-Point* system. This scheme serializes the attack points. It requires the measure be divisible by twelve in some way or another. That can be a time signature of X/3, X/4, X/6 or X/12; or using triplets in a 4/4 bar. Here's one example:



This shows a bar of 6/4 using eighth notes as the base unit. Here's what it looks like with each attack point corresponding to the series integers:



You can only go forward in music, so time-point 5 occurs in the 2nd measure with time-point 6 tied over until it occurs. Yes, it's confusing, to be sure. When I first read *Babbitt's* explanation, I got totally and completely lost. I just couldn't get the reason for tying over to the next time point. It was *Charles Wuorinen* who explained it in a way I finally got. Here's an example of how I used the Time-Point system in one of my own pieces, *Moments in Time*:

Here I established the Time-Point passage in the Violin 1 part, then repeated it in the other strings, staggering the starting point by a measure. What resulted was a sort of canon that added a contrapuntal element to the Time-Point element and, in my mind, made it more interesting. In and by itself, a Time-Point passage may not be compelling enough but, when played alongside of other such passages, becomes so.

You can also not tie each Time-Point to the next and use a shorter duration note value instead. This results in a more *pointillist* feel, similar to some of *Stockhausen's* early works, like *Zeitmass Nr. 5*. You can explore this further by reading *Wuorinen's* and *Babbitt's* writings (see the endnotes for book titles).

The point is math has a place in music, especially modern, post-WWII music. But it's always used in developing schemes ahead of the actual composing process. You don't compose with math; you compose with sounds in time. All music must have some degree of variation within it to keep it interesting to listeners. If it's too repetitive or stagnant, it becomes uninteresting, even boring.

Using some of the mathematical schemes I've described, as well as others of your own devising, is one more way you can introduce variation in a piece and, hopefully, add some interest and complexity to it. It's just another tool, among many others, available to a composer to work with. Like with any tool, for any purpose, you can over-use it, maybe even abuse it, resulting in an outcome you won't be particularly happy with. But using mathematical schemes in your music, with some moderation, can be useful.

How far you want to take it is up to you, and depends on many factors and circumstances. For me, it provides an opportunity to get out of my comfort zone and try something new and different. I find challenging myself now and then keeps things interesting and sharpens my mind. Because your brain will get mushy if you don't challenge it, it's as much therapy as creativity. Now that I'm in my 70s, it's an even better idea.

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[1] The Structure of Atonal Music-Allen Forte-© 1973 Yale University
Basic Atonal Theory-John Rahn-© 1980 Longman, Inc.
Introduction to Post-Tonal Theory-Joseph N. Straus-© 2005 Pearson Education
[2] Xenakis: His Life in Music-James Harley-Taylor & Francis © 2004
Xenakis Matters: Edited by Sharon Kanach-Pendragon Press © 2012
[3] Formalized Music-Iannis Xenakis-Pendragon Press © 1992
[4] Set Theory Objects-Peter Castine-© Peter Lang GmbH © 1994
[5] The Collected Essays of Milton Babbitt-Princeton University Press © 2003
Milton Babbitt: Words about Music-University of Wisconsin Press © 1987
[6] Simple Composition-Charles Wuorinen-C.F. Peters © 1979
[7] http://en.wikipedia.org/wiki/Sonata_form